NORTHERN VALLEY REGIONAL HIGH SCHOOL
Office of Curriculum and Instruction

Mathematics Department
Demarest and Old Tappan

Algebra 1

Unit I	Equations and Inequalities
Unit II	Graphing and Writing Linear Equations and Inequalities
Unit III	Systems of Equations and Inequalities
Unit IV	Regression Models and Linear Functions
Unit V	Functions
Unit VI	Polynomials and Factoring
Unit VII	Irrational Numbers, Quadratic Equations and Quadratic Functions
Unit VIII	Statistics

May 2015

Algebra 1 Unit I - Equations and Inequalities — Time Frame: 3 weeks

Stage 1 – Desired Results

Content Standard(s):

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. *m*

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *m*

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *m*

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *m*

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *m*

Understanding (s)/goals:

Students will understand:

- Solving equations as a reasoning process.
- That the properties of equality guide the process of solving equations.
- The meaning of a solution in context.
- Absolute value as distance.
- Solving inequalities as a reasoning process.
- That the properties of inequality guide the process of solving inequalities.

Essential Question(s):

- How can linear equations in one variable be used to solve real world problems?
- How are the properties of operations and equality of real numbers used to solve equations?
- Why do we use absolute value notation?
- How can absolute value equations be used to solve real world problems?
- How can linear inequalities in one variable be used to solve real world problems?
- How are the properties of operations and inequality of real numbers used to solve equations?

Student objectives (outcomes):

Students will be able to:

- Solve equations with symbols of inclusion and the variable on both sides of the equation. **A-REI.1, 3**
- Distinguish between equations that have no solution, one solution, and infinitely many solutions. **A-REI. 3**
- Create equations to represent real world situations. (include problems with percent increase and decrease) **A-CED.1, 3,**
- Solve literal equations. **A-REI. 3**
- Interpret and solve absolute value equations. **A-REI.1**
- Recognize absolute value equations as two equations in one. **A-REI.1**
- Solve inequalities with symbols of inclusion and the variable on both sides of the inequality. **A-REI.3 A-CED.1**
- Distinguish between inequalities that are true for all real numbers, never true, or have a solution set. **A-REI. 3**
- Create inequalities to represent real world situations. **A-CED 1, 3**

Stage 1 – Desired Results

Content Standard(s):

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (★) *m*

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *m*

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *m*

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *m*

Understanding (s)/goals:	Essential Question(s):
Students will understand: • That the graph of a line is a visual representation of a linear equation in two variables. • That the ordered pairs of points on a line represent the solution set to the corresponding linear equation. • That the solution to a linear inequality is a half-plane. • That the equation of a line can have three different but equivalent forms, each of which conveys specific information about the line.	• What is a linear relation? • Why do we need three different ways of writing the equation of a line? • What does the slope of a line indicate about the line? • What are some types of relationships that can be modeled by linear graphs?

Student objectives (outcomes):

Students will be able to:

- Graph horizontal and vertical lines from their equation.
- Write the equation of horizontal and vertical lines.
- Graph a line by plotting any two points with coordinates that make the equation true.
- Use tables, graphs, equations, and verbal descriptions to describe linear relations.
- Graph lines in standard form Ax + By = C using the x- and y-intercepts. **A.REI.10**
- Solve a linear equation for *y,* and compose a table of values to graph the line. **A.CED.4**
- Identify the *y*-intercept and the slope in the slope-intercept form $y = mx + b$.
- Use the *y*-intercept as one point and the slope to find a second point to graph a line in slope intercept form. **A.REI.10**
- Interpret the slope of a line as the rate of change of *y* relative to a change in *x.*
- Graph lines and appropriately label axes when depicting real-life data; appropriately choose scales for the axes. **A.CED.2**
- Graph the solution to linear inequalities. **A.REI.12**
- Choose *x*- and *y*-intercepts, or slope and y-intercept to graph the boundaries of linear inequalities as appropriate.
- Use the point slope form $y - y_1 = m(x - x_1)$, to write the equation of a line where (x_1, y_1) is a point on the line, and m is the slope. **A.REI.1**

Stage 1 – Desired Results

Content Standard(s):

A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. *a*

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *m*

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*a*

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (★)*m*

Understanding (s)/goals:	Essential Question(s):
Students will understand: • That a linear equation describes a relation between two real variables. • Solving systems as a process of reasoning. • The meaning of the solution to a system in context. • That systems of two equations can be used to solve real life problems. • That the solution obtained through graphing a system of equations is often an approximation. • That visualizing the graphs of the lines helps determine the number of solutions to a system of equations. • That the solution to a system of linear inequalities is the intersection of half-planes.	• Why do we need to solve a system algebraically if we can solve them by graphing? • When and how can we use systems of linear equations to solve real life problem? • When and how can we use systems of linear inequalities to solve real life problem?

Student objectives (outcomes):

Students will be able to:

- Find or approximate the solution to a system of linear equations by graphing. **A.REI.11, A.REI.6**
- Use a graphing calculator to solve a system of linear equations; use the intersect keystroke for increased accuracy. **A.REI.11, A.REI.6**
- Explain graphically what it means for a system of equations to have one solution, no solution, or infinitely many solutions.
- Graph systems of linear inequalities as intersections of half-planes. **A.REI.12**
- Choose *x*- and *y*-intercepts, or slope and y-intercept to graph the boundaries of linear inequalities as appropriate.

- Solve linear equations in two variables by elimination and substitution. **A.REI.5**
- Decide when each method is most effective in solving a system.
- Determine whether a system of equations has one solution, no solution, or infinitely many solutions.
- Create systems of equations to solve real world problems including translation, value, and comparison problems. **A.CED.3**
- Interpret solutions in context. **A.CED.3**

Stage 1 – Desired Results

Content Standard(s):

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (★) *s*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (★) *m*

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (★) *m*

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. (★) *m*

S.ID.9 Distinguish between correlation and causation. (★) *m*

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *m*

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *m*

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *m*

Understanding (s)/goals:

Students will understand:

- That a scatter plot is a visual representation of two related variables.
- That a regression model is a function equation that describes the linear relationship between two variables.
- That the correlation coefficient measures how well the linear model fits the data.
- The meaning of the function notation $f(x)$.
- That $f(a)$ is the y-coordinate of the point on the graph of f when the x-coordinate is a.
- That a function is a rule that assigns a unique value in the range (output) to a given real value in the domain (input).
- Linear functions are used to model variables that change at a constant rate.
- That correlation does not imply causation.

Essential Question(s):

- How are equations of lines used to analyze real data?
- How can linear equations be used to make predictions based on data collected through observations?
- How can linear equations model change over time?

Student objectives (outcomes):

Students will be able to:

- Identify relations that are functions, given a verbal description, an equation, a table of values, or a graph. **F.IF.1, F.IF.2**
- Use the vertical line test. **F.IF.1**
- Use function notation and terminology. **F.IF.1**
- Identify domain and range of a function. **F.IF.2, F.IF.5**
- Verbally describe the rule of the function. **F.IF.2**
- Evaluate functions given an equation, a graph, or a table of values. **F.IF.2**
- Create a scatter plot and estimate the equation of the line of best fit. **S.ID.6**
- Find the regression line by hand and using technology.
- Use technology to find the correlation coefficient and interpret its value in terms of the strength of the linear model. **S.ID.8**
- Use function notation $y = f(x)$ when writing the equation of a line.
- Identify the independent (input) and dependent (output) variables in a linear function. **F.IF.1**
- Model real world applications of lines; identify the independent (input) and dependent (output) variables, and interpret the meaning of the slope and the intercepts in context. **A.CED.2, F.IF.1, S.ID.7**
- Find domain restrictions for linear functions that model real phenomena.
- Use the regression line to make predictions, and explain whether the prediction is reasonable within the context of the problem.
- Distinguish between correlation and causation in a linear relation. **S.ID.9**

Stage 1 – Desired Results
Content Standard(s): **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. *m* **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *m* **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. *m* **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the functions is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. (★) *m* **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *m* **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (★) *m* **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (★) *s* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *s* **F.BF.1** Write a function that describes a relationship between two quantities. (★) *s* **F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *a* **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. (★) *s* a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table.) (★) *s*

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (★) *s*

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. (★) *s*

Understanding (s)/goals:	**Essential Question(s):**
Students will understand: • Function notation • The absolute value function as two functions (piecewise) • The difference between exponential growth and linear growth.	• How can we use functions to model linear and exponential growth?

Student objectives (outcomes):

Students will be able to:

- Make a table of values and graph basic functions: $f(x)=x^2$, $f(x)=x^3$, $f(x)=2^x$, $f(x)= 3^x$. **F.IF.7**
- Compare linear, basic quadratic and simple exponential functions using tables of values, graphs and real data. **F.IF.9, F.LE.1**
- Use graphs and tables to observe that quantities increasing exponentially eventually exceed quantities that follow a linear, quadratic or higher power model. **F.LE.3**
- Write the equation of a linear, quadratic or exponential function give a table of values. **F.BF.1**
- Given a linear model $f(x) = mx + b$ in a real application, interpret the values m and b in context. **F.LE.5**
- Given an exponential model $f(x) = a(b)^x$ in a real application, interpret the values a and b in context. **F.LE.5**
- Make a table of values to graph $f(x) = |x|$. Recognize the absolute value function as a piecewise function. **F.IF.7**
- Graph transformations of the absolute value function. **F.BF.3**
- Identify key features of an absolute value function. **F.IF.4**
- Calculate and interpret the average rate of change of a function over an interval. **F.IF.6**
- Identify an arithmetic sequence as a linear function with positive integers as its domain. **F.IF.3, F.LE.1, F.LE.2, F.LE.3**
- Identify a geometric sequence as an exponential function with positive integers as its domain. **F.IF.3, F.LE.2, F.LE.3**
- Observe that the terms in a geometric sequence eventually exceed terms in an arithmetic sequence. **F.LE.3**

Stage 1 – Desired Results

Content Standard(s):

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *m*

A.SSE.1 Interpret expressions that represent a quantity in terms of its context. (★) *m*
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. *m*

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (★) *s*
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions.

Understanding (s)/goals:	Essential Question(s):
Students will understand: • That polynomials are closed under addition, subtraction and multiplication. • Factoring as a process of changing a sum to a product. • Polynomial form and factored form as equivalent expressions. • Patterns in special products	• How can polynomials be simplified and applied to solve real world problems? • How can we use factoring polynomials in application of real life situations? • What are the patterns in the special products?

Student objectives (outcomes):

Students will be able to:
- Identify parts of a monomial and a polynomial using appropriate terminology. **A.SSE.1a**
- Add, subtract and multiply monomial and polynomial expressions. **A.APR.1**
- Multiply two binomials, and use the patterns $(x – y)(x + y) = x^2 – y^2$ and $(x \pm y)^2 = x^2 \pm 2xy + y^2$. **A.APR.1**
- Simplify or rewrite monomial expression using rules of exponents including power of a power, negative, and zero exponents. **A.SSE.3c**
- Divide a polynomial by a monomial. **A.APR.1, A.SSE.3c**
- Find the greatest common factor for a set of monomials.
- Use the GCF to factor polynomials. **A.SSE.2**
- Factor quadratic trinomials, including trinomials with a leading coefficient other than 1. **A.SSE.2**

- Identify and factor perfect square trinomials. **A.SSE.2**
- Identify and factor difference of two squares binomials. **A.SSE.2**
- Use multiple strategies to factor complex polynomials. **A.SSE.2, A.SSE.3c**

Stage 1 – Desired Results

Content Standard(s):

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *s*

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. *a*

A.REI.4 Solve quadratic equations in one variable. *m*
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (★) *s*
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (★) *m*

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (★) *s*
c. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *s*
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph, and interpret these in terms of a context.

F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *a*

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive

approximations. Include cases where f(x) and/or g(x) are linear with quadratic (calculator only), polynomial, rational, absolute value, exponential, and logarithmic functions. (★)*m*

Understanding (s)/goals:	**Essential Question(s):**
Students will understand: • The arithmetic of irrational numbers. • The properties of sums and products of rational and irrational numbers. • That quadratic equations can be solved by factoring, completing the square, using the quadratic formula, and graphing. • That the real solutions of a quadratic equation are the zeros of a related quadratic function. • The advantages of using the three different but equivalent forms of a quadratic function. • That quadratic functions can be used to model a variety of application problems.	• What is the difference between rational and irrational numbers? • Where does the quadratic formula come from? • How are quadratic models created using real data? • How are quadratic equations used to solve real life problems?

Student objectives (outcomes):

Students will be able to:

- Add, subtract, multiply, divide and simplify irrational numbers. **N.RN.3**
    - Recognize the sum and product of irrational and rational numbers as rational or irrational. **N.RN.3**
- Solve quadratic equations by factoring, completing the square, using the quadratic formula, and graphing a related function. **A.REI.4a,b, A.SSE.3a,b**
- Describe the nature of the solutions and identify quadratic equations that have no real solutions. **A.REI.4b**
- Rewrite the quadratic function in vertex form using completing the square. **A.REI.4**
- Identify transformations and graph quadratic functions in the form $f(x)=a(x-h)^2+k$. **F.BF.3, F.IF.8a**
- Graph quadratic functions (parabolas) in the forms $f(x)=a(x-r)(x-s)$, and $f(x)=ax^2+bx+c$. **A.APR.3, F.IF.8a**
- To find the vertex in any form: use $x=-\frac{b}{2a}$ given the equation in standard form; use the zeros and symmetry in factored form, use (h,k) or transformations in vertex form. **A.SSE.3a,b, F.IF.8a**
- Identify characteristic features of the parabola given its equation in the three different forms. **F.IF.7a, F.IF.8a**
- Use the vertex and the shape of the parabola to describe the range of quadratic functions. **F.IF.7a**
- Find the average rate of change of a quadratic function over a given interval. **F.IF.6**
- Identify maximum or minimum, axis of symmetry, and intervals where the function is increasing or decreasing. **F.IF.7a**
- Create quadratic equations to model and solve real life application problems.
- Solve systems with quadratic functions and linear functions using the graphing calculator. **A.REI.11**

Stage 1 – Desired Results

Content Standard(s):

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). (★) *a*

S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (★) *a*

S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (★) *a*

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (★) *s*

Understanding (s)/goals:

Students will understand:

- The difference between numerical and categorical variables.
- The meaning and interpretation of the measures of center and spread.

Essential Question(s):

- Why is data collected and analyzed?
- How is the way that data is collected, organized and displayed influence interpretation?

Student objectives (outcomes):

Students will be able to:

- Identify a variable as quantitative or categorical.
- Graph numerical data on the real number line, using dot plots, histograms, and box plots. **S.ID.1**
- Use mean, median and mode to compare centers, standard deviation and interquartile range to compare spread, and plots to compare the shape of two or more sets of data. **S.ID.2**
- Identify outliers and explain their effect on the measures of center and spread. **S.ID.3**
- Create a two-way frequency table from two categorical variables; read and interpret data displayed in a two-way table. **S.ID.5**
- Use a two-way frequency table to calculate joint, marginal, and conditional relative frequencies. **S.ID.5**
- Make appropriate displays of joint, marginal, and conditional distributions. **S.ID.5**
- Describe patterns observed in the data. Recognize the association between two variables by comparing conditional and marginal percentages. **S.ID.5**